Measurements of $\sin 2\alpha/\phi_2$ from $B \to \pi\pi$, $\rho\pi$ and $\rho\rho$ modes.

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B meson decays involving $b \to u$ transitions are sensitive to the unitarity triangle angle α (or ϕ_2). The *BABAR* and Belle experiments have studied B-meson decays to $\pi\pi$, $\rho\pi$ and $\rho\rho$ final states. It is possible to combine these measurements to constrain α with a precision of $\mathcal{O}(10°)$ and a central value of approximately 100 degrees. These results are consistent with Standard Model expectations.

1   Introduction

This is a summary of the experimental constraint on α obtained by the B-factories from B meson decays involving $b \to u$ transitions. The final states of interest for these studies are $B \to hh$, where $h = \rho, \pi$. *CP* Violation (CPV) in the Standard Model of particle physics (SM) is described by a single complex phase. Interference between direct decay and decay after B^0-$\overline{B}^0$ mixing results in a time-dependent decay-rate asymmetry between B^0 and $\overline{B}^0$ decaying to h^+h^- that is sensitive to the CKM [1] angle $\alpha \equiv \arg\left[-V_{td}V_{tb}^*/V_{ud}V_{ub}^*\right]$. The presence of loop (penguin) contributions introduces additional phases that can shift the experimentally measurable parameter α_{eff} away from the value of α. In the presence of penguin contributions $\alpha_{\text{eff}} = \alpha + \delta\alpha_{\text{penguin}}$. The CKM angle β is well known and consistent with SM predictions. Any constraint on α constitutes a new test of the SM description of quark mixing and CPV in B-meson decays. Any significant deviation from SM expectation would be a clear indication of new physics. The SM prediction for α are $[95 \pm 9]°$ from [2] and $[98 \pm 16]°$ [3].

There are two classes of measurement that the B-factories are pursuing, the main goal is to use SU(2) relations to relate different hh final states and limit $\delta\alpha_{\text{penguin}}$ in each of the modes $B^0(\overline{B}^0) \to \pi^+\pi^-$, $B^0 \to \rho^+\pi^-$ and $B^0(\overline{B}^0) \to \rho^+\rho^-$. In some cases only weak constraints on α are obtained when using SU(2) analysis and one must use a model dependent approach to obtain a significant result.

Experimental results are quoted with statistical errors preceding systematic errors unless otherwise stated.

2 Isospin analysis of $B \to hh$

One can relate the amplitudes of B decays to $\pi\pi$ final states by noting that $\frac{1}{\sqrt{2}}A^{+-} = A^{+0} - A^{00}$ and $\frac{1}{\sqrt{2}}\overline{A}^{+-} = \overline{A}^{+0} - \overline{A}^{00}$ [4]. These two relations correspond to triangles in a complex plane. In the absence of Electroweak penguin contributions, $|A^{+0}| = |\overline{A}^{+0}|$, i.e. the two triangles have a common base. The phase difference between A^{+-} and $\overline{A}^{+-}$ is related to $|\alpha_{eff} - \alpha|$. Thus, in order to measure α one must measure the branching fractions ($\mathcal{B}$) and charge asymmetries ($\mathcal{A}_{CP}$) of B decays to $\pi^+\pi^-$, $\pi^\pm\pi^0$, $\pi^0\pi^0$. The decays of $B \to \rho\rho$ and $B \to \rho\pi$ are more complicated. In the case of the former decays one has up to three isospin triangle analyses to perform to extract maximal information from data. The final state of B decays to $\rho^\pm\pi^\mp$ is not a CP eigenstate, which results in the need either for a pentagon isospin analysis [5] or a Dalitz Plot (DP) analysis [6].

For initial measurements of α, one can safely neglect electroweak penguin contributions to the isospin analysis as these are a small correction to α [7]. Some estimates of the effect of SU(2) symmetry breaking exist [8], where any correction is estimated to be much less than $10°$.

3 Experimental techniques

Continuum $e^+e^- \to q\bar{q}$ ($q = u, d, s, c$) events are the dominant background. To discriminate signal from continuum one uses either a neural network, Fisher discriminant or likelihood ratio of event shape variables. Signal candidates (B_{rec}) are identified kinematically using two variables, the difference ΔE between the CM energy of the B candidate and $\sqrt{s}/2$, and the beam-energy substituted mass $m_{\mathrm{ES}} = \sqrt{(s/2 + \mathbf{p}_i \cdot \mathbf{p}_B)^2 / E_i^2 - \mathbf{p}_B^2}$, where $\sqrt{s}$ is the total CM energy. The B momentum $\mathbf{p}_B$ and four-momentum of the initial state $(E_i, \mathbf{p}_i)$ are defined in the laboratory frame. The signal is extracted using an extended unbinned maximum likelihood fit.

To study the time-dependent asymmetry one needs to measure the proper time difference, Δt, between the two B decays in the event, and to determine the flavor tag of the other B-meson (B_{tag}). The time difference between the decays of the two neutral B mesons in the event (B_{rec}, B_{tag}) is calculated from the measured separation Δz between the B_{rec} and B_{tag} decay vertices. The B_{rec} vertex is determined from the two charged-pion tracks in its decay. The B_{tag} decay vertex is obtained by fitting the other tracks in the event.

The signal decay-rate distribution $f_+(f_-)$ for $B_{\mathrm{tag}} = B^0$ ($\overline{B}^0$) is given by:

$$f_\pm(\Delta t) = \frac{e^{-|\Delta t|/\tau}}{4\tau}[1 \pm S\sin(\Delta m_d \Delta t) \mp C\cos(\Delta m_d \Delta t)],$$

where τ is the mean B^0 lifetime, Δm_d is the B^0-$\overline{B}^0$ mixing frequency. For B decays to $\rho^+\rho^-$, $S = S_L$ or S_T and $C = C_L$ or C_T are the CP asymmetry parameters for the longitudinal and transversely polarized signal. The decay-rate distribution for B decays to the $\rho^\pm\pi^\mp$ final state is described below. The fitting function takes into account mistag dilution and is convoluted with the Δt resolution function.

CPV is probed by studying the time-dependent decay-rate asymmetry $\mathcal{A} = (R(\Delta t) - \overline{R}(\Delta t))/(R(\Delta t) + \overline{R}(\Delta t))$, where $R(\overline{R})$ is the decay-rate for B^0 ($\overline{B}^0$) tagged events. This

asymmetry has the form $\mathcal{A} = S\sin(\Delta m_d \Delta t) - C\cos(\Delta m_d \Delta t)$[1]. In the case of charged B meson decays (and $\pi^0\pi^0$ as there is no vertex information) one can study a time integrated charge asymmetry, $\mathcal{A}_{CP} = (\overline{N} - N)/(\overline{N} + N)$, where N ($\overline{N}$) is the number of B ($\overline{B}$) decays to the final state. A non-zero measurement of S, C or $\mathcal{A}_{CP}$ for any of the decays understudy would be a clear indication of CP violation.

4 $B \to \pi\pi$

The simplest decays to study in the pursuit of α are $B \to \pi\pi$. Both experiments have measurements for $B \to \pi^+\pi^-$, $\pi^+\pi^0$ and $\pi^0\pi^0$ decays [9, 10]. These results are summarised in Table 1. All of these modes are now well established experimentally and so it is possible to perform an isospin analysis.

mode	Expt	$\mathcal{B}$	S	C
$\pi^+\pi^-$	BABAR	$4.7 \pm 0.6 \pm 0.2$	$-0.30 \pm 0.17 \pm 0.03$	$-0.09 \pm 0.15 \pm 0.04$
	Belle	$4.4 \pm 0.6 \pm 0.3$	$-1.00 \pm 0.21 \pm 0.07$	$-0.58 \pm 0.15 \pm 0.07$

mode	Expt	$\mathcal{B}$	$\mathcal{A}_{CP}$
$\pi^\pm\pi^0$	BABAR	$5.8 \pm 0.6 \pm 0.4$	$-0.01 \pm 0.10 \pm 0.02$
	Belle	$5.0 \pm 1.2 \pm 0.5$	$-0.02 \pm 0.10 \pm 0.01$
$\pi^0\pi^0$	BABAR	$1.17 \pm 0.32 \pm 0.10$	$-0.12 \pm 0.56 \pm 0.06$
	Belle	$2.32^{+0.44}_{-0.48}{}^{+0.22}_{-0.18}$	$-0.43 \pm 0.51^{+0.17}_{-0.16}$

Table 1: Measurements of the $B \to \pi\pi$ decays. BRs are in units of 10^{-6}.

One should note that the Belle measurement of $B^0(\overline{B}^0) \to \pi^+\pi^-$ constitutes an observation of mixing-induced CPV in this decay at a level of 5.2σ, and evidence for direct CPV at a level of 3.2σ. This is the second observation of CPV in B-meson decays and is illustrated in Figure 1 . The BABAR data do not support this conclusion, and the discrepancy between the two experiments is more than 3σ. One must wait for a significant increase in statistics from both experiments to see if the results of the experiments become more compatible.

The $\pi\pi$ isospin analysis is limited by the value of $\mathcal{B}(B^0(\overline{B}^0) \to \pi^0\pi^0)$. Unfortunately this is neither large enough to provide sufficient statistics with the current data set to perform a precision measurement of $\alpha_{eff} - \alpha$, nor small enough to enable a strong bound on this quantity. BABAR have performed an isospin analysis resulting in $|\alpha_{eff} - \alpha| < 35°$ (90% C.L.). It is possible to derive model dependent constraints on α, as done by the Belle Collaboration; $90 < \alpha < 146°$ [10] (90% C.L.) using [11].

5 $B \to \rho\rho$

The decays of $B \to \rho\rho$ correspond to a spin zero particle decaying into two spin one, vector (V), particles. As a result, the CP analysis of B decays to $\rho^+\rho^-$ is complicated by the presence

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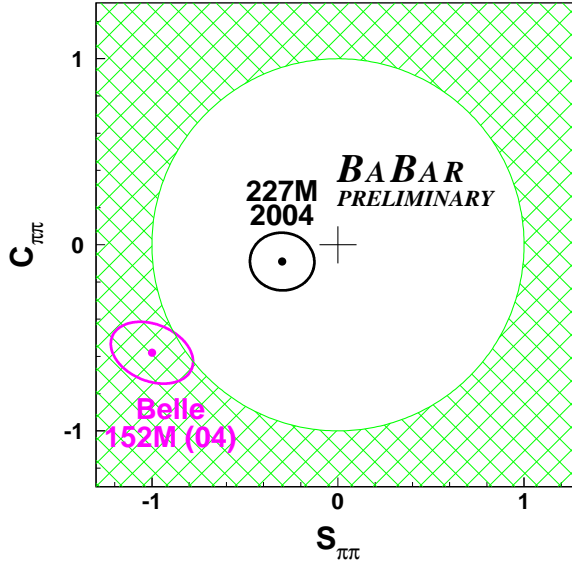[1]Belle use a different convention with $C = -\mathcal{A}_{CP}$

Figure 1: A plot of the $S_{\pi\pi}$ vs $C_{\pi\pi}$ plane for the results from *BABAR* and Belle. The hatched region is unphysical.

of three helicity states ($h = 0, \pm1$). The $h = 0$ state corresponds to longitudinal polarization and is CP-even, while neither the $h = +1$ nor the $h = -1$ state is an eigenstate of CP. The longitudinal polarization fraction f_L is defined as the fraction of the helicity zero state in the decay. The angular distribution is

$$\frac{d^2\Gamma}{\Gamma d\cos\theta_1 d\cos\theta_2} = \frac{9}{4}\left(f_L\cos^2\theta_1\cos^2\theta_2 + \frac{1}{4}(1-f_L)\sin^2\theta_1\sin^2\theta_2\right) \qquad (1)$$

where $\theta_i, i = 1,2$ is defined for each ρ meson as the angle between the π^0 momentum in the ρ rest frame and the flight direction of the B^0 in this frame. The angle between the ρ-decay planes is integrated over to simplify the analysis. A full angular analysis of the decays is needed in order to separate the definite CP contributions; if however a single CP channel dominates the decay (which has been experimentally verified), this is not necessary [12]. The longitudinal polarization dominates this decay [13, 14]. Not all of the $\rho\rho$ final states have been observed, however as $\mathcal{B}(B^0 \to \rho^0\rho^0)$ is small, one can conservatively assume that is longitudinally dominated when performing an isospin analysis. There are assumptions in addition to SU(2) that are made when performing a $\rho\rho$ isospin analysis. These are (i) neglect Electroweak penguin contributions [a few ° effect] (ii) ignore possible $I = 1$ components in the decay [15][2]. The measured branching fractions, f_L, S and C for $B \to \rho\rho$ are summarised in Table 2.

Recent measurements of the $B^+ \to \rho^+\rho^0$ branching fraction and upper limit for $B^0 \to \rho^0\rho^0$ [16] indicate small penguin contributions in $B \to \rho\rho$, as has been found in some calculations [17]. The decay mode $B^\pm \to \rho^\pm\rho^0$ has only been studied using a small fraction of the available data of the B-factories. As has been remarked, it is important to see this mode updated in the near future [2, 3].

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[2]This is a reasonable assumption given available statistics.

mode	$\mathcal{B}$	f_L	S	C
$\rho^+\rho^-$	$30 \pm 5 \pm 4$	$0.99 \pm 0.03 \pm 0.04$	$-0.19 \pm 0.33 \pm 0.11$	$-0.23 \pm 0.24 \pm 0.14$

mode	Expt	$\mathcal{B}$	f_L	$\mathcal{A}_{CP}$
$\rho^\pm\rho^0$	BABAR	$22.5^{+5.7}_{-5.4} \pm 5.8$	$0._{-0.07}^{+0.05}$	$-0.19 \pm 0.23 \pm 0.03$
$\rho^\pm\rho^0$	Belle	$31.7 \pm 7.1^{+3.8}_{-6.7}$	$0._{-0.07}^{+0.05}$	$0.00 \pm 0.22 \pm 0.03$
$\rho^0\rho^0$	BABAR	< 1.1 (90% C.L.)	-	-

Table 2: Measurements of the $B \to \rho\rho$ decays. BRs are in units of 10^{-6}. $f_L=1$ is assumed in order to calculate the upper limit on $\mathcal{B}(B^0(\overline{B}^0) \to \rho^0\rho^0)$.

Given that penguin pollution is small, it is possible to perform an isospin analysis of the longitudinal polarisation of the $B \to \rho\rho$ decays, and use the results of BABAR's time-dependent CP analysis of $\rho^+\rho^-$ [14] to constrain α. If one does this, using the afforementioned assumptions, one obtains $\alpha = (96 \pm 10(\text{stat}) \pm 5(\text{syst}) \pm 11(\text{penguin}))°$. The "penguin" error is determined primarily from the experimental knowledge of the $\rho^0\rho^0$ and $\rho^\pm\rho^0$ branching ratios. Figure 2 shows the confidence level plot of α corresponding to the isospin analysis of the longitudinally polarized $\rho\rho$ data.
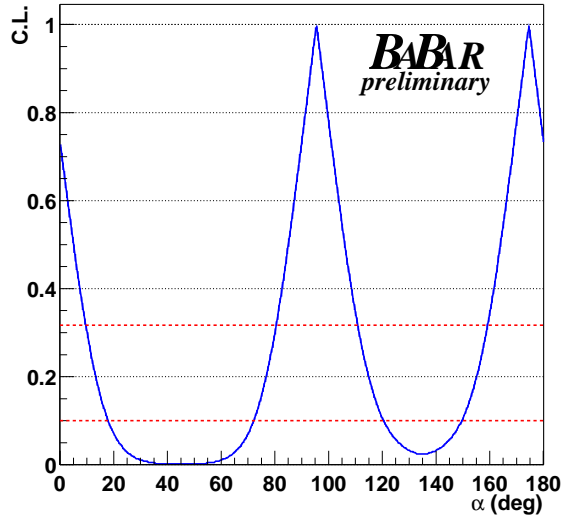

Figure 2: A plot of the confidence level of α determined from BABAR's $\rho\rho$ isospin analysis.

6 $B \to \rho\pi$

The decays $B \to \rho\pi$ can be analysed in two different ways. The more straight forward approach is to cut away interference regions of the $\pi^+\pi^-\pi^0$ DP and analyse the regions in the vicinity of ρ resonances. This is called the Quasi-2-body approach (Q2B) and it avoids the need to

understand the interference regions. The drawback of the Q2B method is that one looses statistical power by cutting on the DP. A corollary of this is that one requires more statistics than the B-factories currently have in order to get a significant constraint from the pentagon isospin analysis. The alternative is to perform an analysis of the $B \to \pi^+\pi^-\pi^0$ DP, accounting for the interference between intersecting ρ resonance bands and other resonant structure. Both of these approaches have been studied by *BABAR* [18]. Belle has performed a Q2B analysis of $\rho\pi$ [19], and the first time-dependent DP analysis has been performed by *BABAR* [20].

In the Q2B approach, one fits a time-dependence of

$$f_\pm(\Delta t) = (1 \pm \mathcal{A}_{CP})\frac{e^{-|\Delta t|/\tau}}{4\tau}[(S \pm \Delta S \sin(\Delta m_d \Delta t) - (C \pm \Delta C)\cos(\Delta m_d \Delta t)],$$

where there are three additional parameters, a charged asymmetry, $\mathcal{A}_{CP}$, between decays to $\rho^+\pi^-$ and $\rho^-\pi^+$ final states, and two dilution parameters; ΔS and ΔC. The details of the DP analysis can be found in [20], where one varies a larger number of parameters in the nominal fit, and converts these to the same observables as the Q2B approach. Table 3 summarises the experimental constraints on $B \to \rho\pi$ (DP analysis from *BABAR* and Q2B analysis from Belle).

mode	Expt	S	C	$\mathcal{A}_{CP}$
$B \to \pi^+\pi^-\pi^0$	*BABAR*	$-0.10 \pm 0.14 \pm 0.04$	$0.34 \pm 0.11 \pm 0.05$	$-0.088 \pm 0.049 \pm 0.013$
$B^0 \to \rho^+\pi^-$	Belle	$-0.28 \pm 0.23^{+0.10}_{-0.08}$	$0.25 \pm 0.17^{+0.02}_{-0.06}$	$-0.16 \pm 0.10 \pm 0.02$

Table 3: Measurements of the $B \to \rho\rho$ decays. BRs are in units of 10^{-6}.

Using SU(2) with *BABAR*'s DP result, one obtains the following constraint; $\alpha = (113^{+27}_{-17}\pm6)°$ (Figure 3 shows the corresponding confidence level plot for α). This result is self consistent as the strong phase differences and amplitudes are determined solely from the structure of the DP. The unique aspect of this result is that there is only a single solution between 0 and 180°, thus a two fold ambiguity in the $\rho\eta$ plane. As a result this measurement is an important constraint on alpha, ancillary to that of the *BABAR* $B^0 \to \rho\rho$ analysis.

It is possible to derive a model dependent constraint on alpha from these data. As shown by Gronau and Zupan [21]. One can make several assumptions, including (i) the relative strong phase differences between tree level contributions for $\rho^+\pi^-$ and $\rho^-\pi^+$ being less than 90° (ii) SU(3) being exact for penguin amplitudes (iii) neglecting Electroweak penguins (iv) neglecting annihilation and exchange diagrams to derive a constraint on α. SU(3) breaking effects are estimated in this model using CLEO, *BABAR* and Belle data. When doing this Belle obtain $\alpha = (102 \pm 11_{expt} \pm 15_{model})°$. Gronau and Zupan compute the corresponding result for the *BABAR* analysis as $(93 \pm 4_{expt} \pm 15_{model})°$. There are other models proposed in the literature, for example [22].

7   Comparison with the Standard Model

The indirect measurements of α from the SM are $95 \pm 9°$ [2] and $98 \pm 16°$ [3] from the UT Fit and CKM Fitter groups, respectively. Direct measurements from the B-factories are in
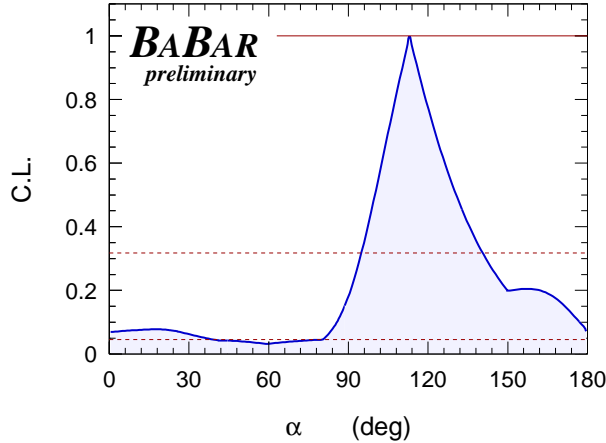
Figure 3: A plot of the confidence level of α determined from *BABAR*'s $\rho\pi$ DP analysis.

good agreement with these predictions, where the average of α from hh decays, using isospin, is known to $\mathcal{O}(10°)$. The precision of this result is dominated by $B \to \rho\rho$, with an important contribution from $B \to \rho\pi$, where the latter reduces this constraint from four-fold to a two-fold ambiguity in the $\rho\eta$ plane. In the next few years α should be known to $\mathcal{O}(2-3°)$ and the B-factories will be able to cleanly test the closure of unitarity triangle with precision for the first time. A significant isospin analysis result from $B \to \pi\pi$ is still some way off. Similarly, the model dependent calculations of α are in agreement with the SM.

8 Summary

In the past year the B-factories have made significant progress towards a precision measurement of the unitarity triangle angle α, only relying on SU(2) symmetry. The dominant constraint comes from $B \to \rho\rho$ decays [14], which has a four-fold ambiguity in the $\rho\eta$ plane. The *BABAR* analysis of $B \to \pi^+\pi^-\pi^0$ decays represents the first time-dependent *CP* analysis. Inclusion of the interference regions of the DP enables one to measure strong phase differences between amplitudes contributing to the final state; In turn this enables one to derive a self consistent constraint of α with a two fold ambiguity in the $\rho\eta$ plane. One can also employ models to further constrain α as has been discussed herein.

8.1 Bibliography

Acknowledgments

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